Tables for Asymmetric Multi-Element Coupled-Transmission-Line Directional Couplers

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Summary—**Tables are presented for the design of TEM-mode asymmetric multi-element coupled-transmission-line directional couplers, having a very broad-band Chebyshev equal-ripple coupling response. Coupler designs of 2 to 6 elements having bandwidths up to 20:1 and mean couplings of 3, 6, 10, 15 and 20 db are tabulated.**

INTRODUCTION

THE MULTI-ELEMENT asymmetric coupled-transmission-line directional coupler is shown diagrammatically in Fig. 1. It consists of a cascaded set of coupled TEM-mode transmission lines, each of identical electrical length. The individual coupling of each section becomes progressively looser proceeding from left to right in Fig. 1, *i.e.*, the coupling is a stepped monotonic function of distance along the coupling region.

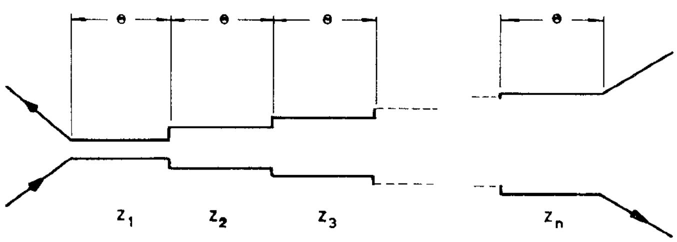

Fig. 1—Even-mode impedances for the n-element coupler.

The main features of this class of directional couplers, which it has in common with other TEM-mode couplers, are that the input VSWR and isolation are theoretically perfect at all frequencies. The coupling may be expressed as a simple function of frequency, and the structure is therefore amenable to synthesis by modern network techniques. In consequence, optimum Chebyshev equal-ripple coupling functions are realizable with extremely wide-band coupling characteristics, without necessarily requiring a very large number of coupled sections. On the other hand, since the coupler is asymmetric, the phase difference between the coupled waves at the ends of the coupler region is not 90° at all frequencies, as is the case with any symmetrical coupler. By choosing suitable reference planes, however, 90°-phase difference may be obtained approximately.[1]

The coupling of each section is defined by the even and odd mode impedances Z_{oe} and Z_{oo}, respectively.[2] In the present context these impedances are normalized with respect to the characteristic impedance of the input ports, and the relationship

$$Z_{oe}Z_{oo} = 1 \tag{1}$$

holds for each coupled element. Eq. (1) ensures the perfect input VSWR and isolation of the coupler. In consequence of (1), any individual coupled element is completely specified by its even-mode impedance Z_{oe}. Since $Z_{oe} > Z_{oo}$, $Z_{oe} > 1$. The coupling is determined by the ratio

$$Z_{oe}/Z_{oo} = Z_{oe}^2$$

which approaches unity for very loose coupling, so that the looser the coupling, the smaller is Z_{oe}. Suppressing the *oe* suffices, the several even-mode impedances of the n elements may be specified by the monotonically decreasing series $Z_1, Z_2, \cdots, Z_n$ as shown in Fig. 1, in which every value of Z_r is greater than unity.

THE COMPUTATION

In a previous paper,[1] exact closed formulas were given for the even-mode impedances of the elements of asymmetric coupled-transmission-line directional couplers for 2 to 5 elements in terms of the bandwidth, coupling and ripple. The formulas are basically quite simple but require a considerable amount of computation to derive even one design. It was decided to prepare a set of tables from which any coupler may be designed rapidly. A computer program was written which derives the even-mode impedances directly by synthesis rather than by using the actual closed formulas. By supplying data giving the number of elements, the mean coupling and the bandwidth, the corresponding ripple is computed and printed with the derived even-mode impedances. Thus it is a general program and will give results for almost any number of elements, n, although the present computation gives results to only 6 elements. A limitation on n will occur because the accuracy of the

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[1] R. Levy, "General synthesis of asymmetric multi-element coupled-transmission-line directional couplers," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-11, pp. 226–237; July, 1963.

[2] E. M. T. Jones and J. T. Bolljahn, "Coupled strip-transmission-line filters and directional couplers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 75–81; April, 1956.

computer is limited by the effective number of decimal places to which numbers are held, 9 in the present case (*i.e.*, 30 binary digits). Synthesis for large values of n would demand better than 9-figure accuracy. A more accurate computer might then be used, but this was not justified in the present instance because generally only a very small number of elements is required. It is considered that the present program is capable of deriving couplers to orders higher than $n=6$ if greater bandwidths or other applications were to create a demand for this.

Use of Tables

The coupling characteristic is shown in Fig. 2, which defines the mean coupling C and the ripple R, both in units of decibels. The edges of the pass band are at frequencies f_1 and f_2, and the bandwidth is defined as the ratio

$$BW = f_2/f_1.$$

Tables I–V (pages 276–279) give the normalized even-mode impedances for 2- to 6-element couplers. Sufficient results are given for accurate interpolation.

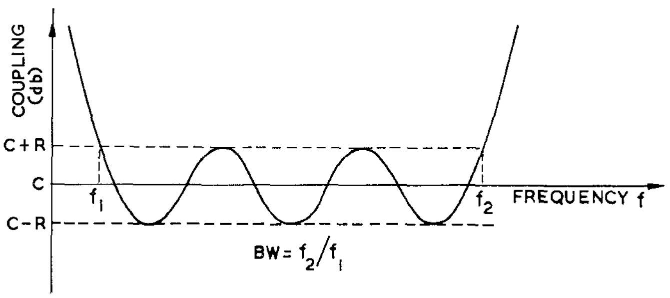

Fig. 2—Directional coupler characteristic.

It is interesting to compare the tabulated results with the examples given in the earlier paper,[1] which were calculated by logarithms and a desk calculator. The comparison is shown in Table VI (page 279), and indicates very close agreement. The inconsistencies in the values of R are due to errors in reading from a graph of R vs BW. Minor differences in the impedances are due to errors in the earlier calculations.

The impedances may be difficult to realize in practice for the case of tight coupling, *e.g.*, a 6-element 3-db coupler for a 20:1 bandwidth requires $Z_1=4.9$. With an input impedance of 50 ohms, this gives

$$Z_{oe} = 245 \text{ ohms}, \qquad Z_{oo} = 10.2 \text{ ohms}.$$

These values are impossible to realize by normal techniques, but the introduction of re-entrant cross-sectional coupled line[3] enables impedances of this order to be obtained with reasonable dimensions. In the above example, the re-entrant line impedances are given by

$$Z_{01} = \tfrac{1}{2}(Z_{oe} - Z_{oo}) = 117.4 \text{ ohms}$$

$$Z_{02} = Z_{oo} = 10.2 \text{ ohms}$$

i.e., this section would consist of a 117.4-ohm line containing two 10.2-ohm lines within its inner conductor. While this would be feasible at lower microwave frequencies, it would still present great difficulties at an upper working frequency of, say, 5 Gc per second, because of the limitation on the over-all size of the structures required by the necessity to prevent severe discontinuity effects[3] and propagation of waveguide type modes.

[3] S. B. Cohn, "The re-entrant cross section and wide-band 3-db hybrid couplers," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 254–258; July, 1963.

TABLE I
Normalized Even-Mode Impedances of Two-Element Couplers

$N=2$, Coupling = 3.00 db						
BW	2.0	3.0	4.0	5.0	6.0	7.0
R (db)	0.022	0.128	0.292	0.480	0.673	0.858
Z_1	2.9655	3.1292	3.2895	3.4455	3.6007	3.7574
Z_2	1.2308	1.3206	1.4235	1.5320	1.6434	1.7563

$N=2$, Coupling = 6.00 db						
BW	2.0	3.0	4.0	5.0	6.0	7.0
R (db)	0.034	0.191	0.439	0.724	1.017	1.303
Z_1	1.9866	2.0596	2.1295	2.1958	2.2603	2.3240
Z_2	1.1481	1.2046	1.2680	1.3335	1.3994	1.4648

$N=2$, Coupling = 10.0 db						
BW	2.0	3.0	4.0	5.0	6.0	7.0
R (db)	0.040	0.230	0.528	0.871	1.226	1.575
Z_1	1.5073	1.5412	1.5729	1.6025	1.6307	1.6581
Z_2	1.0882	1.1210	1.1573	1.1940	1.2302	1.2655

$N=2$, Coupling = 15.0 db						
BW	2.0	3.0	4.0	5.0	6.0	7.0
R (db)	0.043	0.248	0.568	0.938	1.321	1.698
Z_1	1.2533	1.2688	1.2833	1.2966	1.3091	1.3211
Z_2	1.0480	1.0656	1.0847	1.1038	1.1232	1.1402

$N=2$, Coupling = 20.0 db						
BW	2.0	3.0	4.0	5.0	6.0	7.0
R (db)	0.044	0.253	0.580	0.959	1.351	1.737
Z_1	1.1344	1.1423	1.1496	1.1562	1.1624	1.1684
Z_2	1.0266	1.0362	1.0466	1.0569	1.0668	1.0763

TABLE II
NORMALIZED EVEN-MODE IMPEDANCES OF THREE-ELEMENT COUPLERS

$N=3$, Coupling = 3.00 db							
BW	4.0	5.0	6.0	7.0	8.0	9.0	10.0
R (db)	0.076	0.160	0.266	0.385	0.509	0.635	0.758
Z_1	3.5255	3.6516	3.7672	3.8765	3.9828	4.0881	4.1938
Z_2	1.6386	1.7528	1.8633	1.9072	2.0742	2.1759	2.2759
Z_3	1.1382	1.1925	1.2520	1.3149	1.3801	1.4469	1.5149
$N=3$, Coupling = 6.00 db							
BW	4.0	5.0	6.0	7.0	8.0	9.0	10.0
R (db)	0.114	0.241	0.400	0.579	0.768	0.959	1.149
Z_1	2.2239	2.2752	2.3215	2.3646	2.4058	2.4462	2.4862
Z_2	1.3875	1.4510	1.5112	1.5683	1.6229	1.6754	1.7263
Z_3	1.0920	1.1276	1.1661	1.2063	1.2475	1.2891	1.3308
$N=3$, Coupling = 10.0 db							
BW	4.0	5.0	6.0	7.0	8.0	9.0	10.0
R (db)	0.137	0.290	0.482	0.697	0.925	1.156	1.387
Z_1	1.6140	1.6364	1.6564	1.6747	1.6921	1.7089	1.7253
Z_2	1.2215	1.2555	1.2870	1.3166	1.3443	1.3707	1.3959
Z_3	1.0559	1.0772	1.1000	1.1235	1.1473	1.1710	1.1945
$N=3$, Coupling = 15.0 db							
BW	4.0	5.0	6.0	7.0	8.0	9.0	10.0
R (db)	0.147	0.312	0.518	0.750	0.996	1.246	1.495
Z_1	1.3016	1.3115	1.3203	1.3283	1.3359	1.3431	1.3501
Z_2	1.1174	1.1346	1.1503	1.1649	1.1785	1.1912	1.2033
Z_3	1.0308	1.0424	1.0547	1.0672	1.0798	1.0922	1.1044
$N=3$, Coupling = 20.0 db							
BW	4.0	5.0	6.0	7.0	8.0	9.0	10.0
R (db)	0.150	0.319	0.530	0.767	1.018	1.274	1.529
Z_1	1.1578	1.1636	1.1680	1.1719	1.1756	1.1792	1.1826
Z_2	1.0642	1.0733	1.0816	1.0892	1.0963	1.1030	1.1092
Z_3	1.0172	1.0236	1.0303	1.0372	1.0440	1.0507	1.0573

TABLE III
NORMALIZED EVEN-MODE IMPEDANCES OF FOUR-ELEMENT COUPLERS

$N=4$, Coupling = 3.00 db						
BW	4.0	6.0	8.0	10.0	12.0	14.0
R (db)	0.020	0.105	0.250	0.427	0.613	0.796
Z_1	3.7032	3.9210	4.0974	4.2556	4.4088	4.5637
Z_2	1.8149	2.0471	2.2520	2.4379	2.6121	2.7792
Z_3	1.2407	1.3789	1.5184	1.6556	1.7904	1.9232
Z_4	1.0502	1.1108	1.1881	1.2753	1.3683	1.4648
$N=4$, Coupling = 6.00 db						
BW	4.0	6.0	8.0	10.0	12.0	14.0
R (db)	0.030	0.158	0.376	0.642	0.925	1.208
Z_1	2.2934	2.3787	2.4459	2.5048	2.5608	2.6166
Z_2	1.4814	1.6028	1.7056	1.7961	1.8786	1.9560
Z_3	1.1561	1.2415	1.3249	1.4047	1.4810	1.5546
Z_4	1.0340	1.0749	1.1262	1.1831	1.2426	1.3031
$N=4$, Coupling = 10.0 db						
BW	4.0	6.0	8.0	10.0	12.0	14.0
R (db)	0.036	0.190	0.452	0.773	1.115	1.459
Z_1	1.6438	1.6802	1.7083	1.7326	1.7553	1.7775
Z_2	1.2705	1.3328	1.3840	1.4279	1.4671	1.5032
Z_3	1.0933	1.1424	1.1891	1.2327	1.2735	1.3120
Z_4	1.0210	1.0459	1.0768	1.1105	1.1451	1.1796
$N=4$, Coupling = 15.0 db						
BW	4.0	6.0	8.0	10.0	12.0	14.0
R (db)	0.038	0.204	0.487	0.832	1.201	1.572
Z_1	1.3147	1.3306	1.3428	1.3532	1.3629	1.3722
Z_2	1.1419	1.1726	1.1973	1.2181	1.2364	1.2531
Z_3	1.0509	1.0769	1.1012	1.1235	1.1441	1.1632
Z_4	1.0117	1.0254	1.0423	1.0604	1.0788	1.0968
$N=4$, Coupling = 20.0 db						
BW	4.0	6.0	8.0	10.0	12.0	14.0
R (db)	0.039	0.209	0.497	0.851	1.229	1.608
Z_1	1.1652	1.1730	1.1790	1.1841	1.1888	1.1933
Z_2	1.0771	1.0932	1.1060	1.1168	1.1261	1.1346
Z_3	1.0282	1.0424	1.0555	1.0675	1.0784	1.0884
Z_4	1.0065	1.0142	1.0235	1.0335	1.0435	1.0533

TABLE IV
NORMALIZED EVEN-MODE IMPEDANCES OF FIVE-ELEMENT COUPLERS

			$N=5$, Coupling = 3.00 db				
BW	6.0	8.0	10.0	12.0	14.0	16.0	18.0
R (db)	0.042	0.123	0.239	0.377	0.524	0.673	0.818
Z_1	4.0463	4.2052	4.3384	4.4595	4.5767	4.6945	4.8155
Z_2	2.2021	2.4053	2.5831	2.7439	2.8936	3.0365	3.1751
Z_3	1.4917	1.6410	1.7815	1.9147	2.0424	2.1660	2.2869
Z_4	1.1831	1.2761	1.3726	1.4703	1.5682	1.6659	1.7633
Z_5	1.0510	1.0980	1.1557	1.2206	1.2903	1.3631	1.4381
			$N=5$, Coupling = 6.00 db				
BW	6.0	8.0	10.0	12.0	14.0	16.0	18.0
R (db)	0.062	0.184	0.360	0.567	0.790	1.017	1.241
Z_1	2.4247	2.4839	2.5326	2.5762	2.6178	2.6592	2.7012
Z_2	1.6779	1.7760	1.8592	1.9326	1.9995	2.0622	2.1221
Z_3	1.3064	1.3920	1.4702	1.5423	1.6099	1.6741	1.7356
Z_4	1.1208	1.1800	1.2400	1.2995	1.3580	1.4152	1.4713
Z_5	1.0349	1.0668	1.1055	1.1484	1.1939	1.2406	1.2880
			$N=5$, Coupling = 10.0 db				
BW	6.0	8.0	10.0	12.0	14.0	16.0	18.0
R (db)	0.075	0.222	0.432	0.682	0.952	1.226	1.499
Z_1	1.6992	1.7237	1.7436	1.7612	1.7778	1.7941	1.8104
Z_2	1.3696	1.4173	1.4568	1.4911	1.5218	1.5501	1.5767
Z_3	1.1781	1.2248	1.2664	1.3040	1.3386	1.3709	1.4013
Z_4	1.0730	1.1077	1.1422	1.1759	1.2083	1.2395	1.2696
Z_5	1.0216	1.0411	1.0646	1.0903	1.1171	1.1443	1.1715
			$N=5$, Coupling = 15.0 db				
BW	6.0	8.0	10.0	12.0	14.0	16.0	18.0
R (db)	0.081	0.238	0.465	0.735	1.025	1.321	1.616
Z_1	1.3388	1.3494	1.3579	1.3654	1.3723	1.3791	1.3859
Z_2	1.1902	1.2129	1.2315	1.2473	1.2614	1.2742	1.2862
Z_3	1.0954	1.1193	1.1402	1.1589	1.1758	1.1914	1.2060
Z_4	1.0401	1.0587	1.0770	1.0945	1.1113	1.1272	1.1424
Z_5	1.0120	1.0228	1.0357	1.0496	1.0640	1.0784	1.0927
			$N=5$, Coupling = 20.0 db				
BW	6.0	8.0	10.0	12.0	14.0	16.0	18.0
R (db)	0.082	0.244	0.476	0.751	1.048	1.351	1.653
Z_1	1.1771	1.1822	1.1864	1.1900	1.1934	1.1967	1.1999
Z_2	1.1024	1.1141	1.1236	1.1316	1.1387	1.1452	1.1512
Z_3	1.0524	1.0652	1.0763	1.0861	1.0950	1.1031	1.1106
Z_4	1.0223	1.0325	1.0425	1.0520	1.0610	1.0695	1.0775
Z_5	1.0067	1.0127	1.0199	1.0276	1.0355	1.0433	1.0511

TABLE V

NORMALIZED EVEN-MODE IMPEDANCES OF SIX-ELEMENT COUPLERS

$N=6$, Coupling = 3.00 db

BW	8.0	10.0	12.0	14.0	16.0	18.0	20.0
R (db)	0.060	0.134	0.232	0.344	0.465	0.588	0.711
Z_1	4.2976	4.4178	4.5218	4.6176	4.7105	4.8038	4.8992
Z_2	2.5378	2.7112	2.8640	3.0027	3.1320	3.2551	3.3743
Z_3	1.7504	1.8944	2.0274	2.1520	2.2704	2.3841	2.4944
Z_4	1.3572	1.4618	1.5639	1.6635	1.7606	1.8559	1.9496
Z_5	1.1540	1.2224	1.2946	1.3687	1.4441	1.5200	1.5962
Z_6	1.0523	1.0905	1.1363	1.1875	1.2426	1.3005	1.3603

$N=6$, Coupling = 6.00 db

BW	8.0	10.0	12.0	14.0	16.0	18.0	20.0
R (db)	0.090	0.201	0.348	0.518	0.700	0.888	1.077
Z_1	2.5164	2.5596	2.5965	2.6301	2.6623	2.6944	2.7270
Z_2	1.8357	1.9145	1.9821	2.0424	2.0976	2.1494	2.1988
Z_3	1.4506	1.5280	1.5976	1.6613	1.7206	1.7766	1.8301
Z_4	1.2284	1.2913	1.3512	1.4084	1.4632	1.5160	1.5672
Z_5	1.1026	1.1469	1.1928	1.2393	1.2857	1.3318	1.3773
Z_6	1.0360	1.0620	1.0929	1.1271	1.1635	1.2012	1.2397

$N=6$, Coupling = 10.0 db

BW	8.0	10.0	12.0	14.0	16.0	18.0	20.0
R (db)	0.109	0.242	0.418	0.623	0.843	1.070	1.299
Z_1	1.7368	1.7543	1.7691	1.7825	1.7952	1.8077	1.8203
Z_2	1.4452	1.4820	1.5130	1.5403	1.5650	1.5878	1.6094
Z_3	1.2554	1.2958	1.3313	1.3633	1.3926	1.4198	1.4455
Z_4	1.1350	1.1704	1.2035	1.2346	1.2639	1.2916	1.3182
Z_5	1.0624	1.0887	1.1155	1.1423	1.1686	1.1944	1.2196
Z_6	1.0223	1.0383	1.0571	1.0778	1.0994	1.1216	1.1440

$N=6$, Coupling = 15.0 db

BW	8.0	10.0	12.0	14.0	16.0	18.0	20.0
R (db)	0.117	0.261	0.450	0.670	0.907	1.153	1.400
Z_1	1.3550	1.3624	1.3687	1.3743	1.3796	1.3848	1.3900
Z_2	1.2259	1.2430	1.2573	1.2697	1.2808	1.2910	1.3005
Z_3	1.1346	1.1547	1.1721	1.1876	1.2016	1.2145	1.2266
Z_4	1.0731	1.0915	1.1086	1.1244	1.1392	1.1530	1.1661
Z_5	1.0343	1.0486	1.0629	1.0771	1.0909	1.1043	1.1172
Z_6	1.0124	1.0213	1.0316	1.0429	1.0546	1.0664	1.0783

$N=6$, Coupling = 20.0 db

BW	8.0	10.0	12.0	14.0	16.0	18.0	20.0
R (db)	0.119	0.266	0.460	0.685	0.928	1.179	1.431
Z_1	1.1850	1.1886	1.1916	1.1943	1.1969	1.1994	1.2019
Z_2	1.1207	1.1294	1.1366	1.1429	1.1484	1.1535	1.1583
Z_3	1.0733	1.0839	1.0930	1.1011	1.1083	1.1150	1.1211
Z_4	1.0403	1.0503	1.0595	1.0679	1.0758	1.0831	1.0899
Z_5	1.0191	1.0270	1.0348	1.0425	1.0500	1.0572	1.0642
Z_6	1.0070	1.0119	1.0176	1.0239	1.0303	1.0368	1.0433

TABLE VI

COMPARISON OF RESULTS OBTAINED BY PREVIOUS COMPUTATION[1] WITH THE COMPUTER RESULTS

N	2		2		2		3	
BW	4		4		6.677		6	
C (db)	10		20		3.235		20	
	Results[1]	Computer	Results[1]	Computer	Results[1]	Computer (interpolated)	Results[1]	Computer
R (db)	0.5	0.528	0.6	0.580	0.835	0.832	0.5	0.530
Z_1	1.562	1.5729	1.143	1.1496	3.52	3.59	1.164	1.1680
Z_2	1.154	1.1573	1.046	1.0466	1.69	1.70	1.080	1.0816
Z_3	—	—	—	—	—	—	1.030	1.0303